GREEDY ALGORITHMS

— Observation that make a problem simpler.

$N \geq 3$, $(A_1, A_2, \ldots, A_N)$; $i < j < k$

$(A_i + A_j + A_k)$ is minimum

"Greedily" choosing the 3 minimum elements.

$O(N^3) \longrightarrow O(N)$

$$A_i, \; A_j, \; A_k$$

$$(A_{i'} + A_j + A_{k'}) \longrightarrow$$

$$A_{j'} \geq \max\{A_i, A_j, A_k\}$$

$$j' \notin \{i, j, k\}$$

MINIMIZE THE VALUE (CDC Series 10)

https://www.hackerrank.com/contests/goc-cdc-series-10/challenges/minimize-the-value-1/problem

$A_1, A_2, \ldots, A_N$,

$N \leq 10^5$

$-10^9 \leq A_i \leq 10^9$

$i < j$

$\left\{ \frac{A_j - A_i}{(j - i)} \right\}$

$\underline{O(N^2)} \longrightarrow$ Check all pairs (i, j)

Sug. 1: sort acc. to $\langle A_i, i \rangle$, check consecutive pairs.

Sug. 2: $j \rightarrow A_j$ min

$i \rightarrow A_i$ max.

$N = 3$, $A = \{6, 5, 1\}$

$A = \{7, 6, 8, 1\}$, (-7)

1 2 3 4

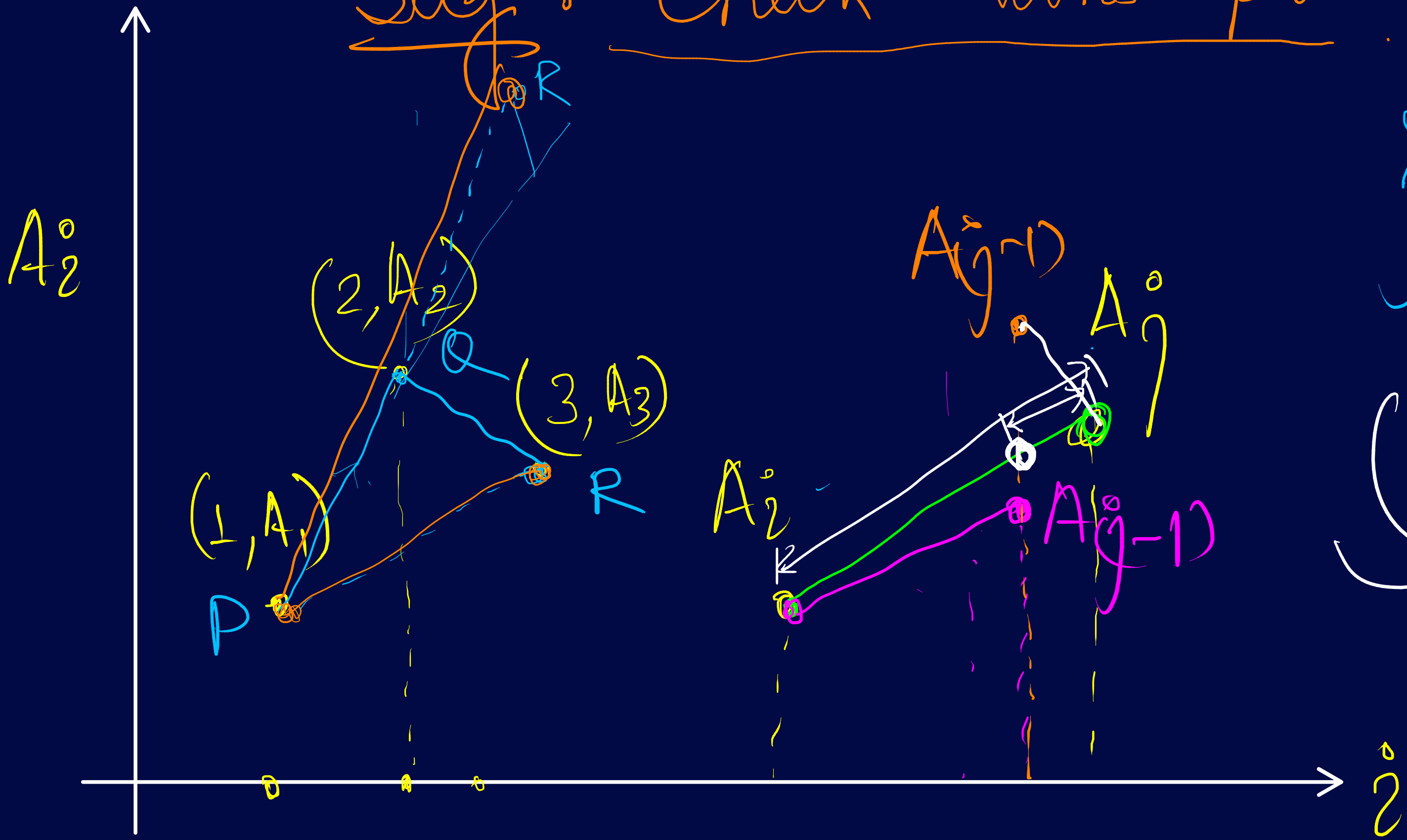
min value

$= \{(1, 4), (6, 2), (7, 1), (8, 3)\}$

(-1)

$(-5/2)$ $(+1/2)$

Sug: Check cons. pairs



$j = \{i+1\}$

$\underbrace{(j-1, j)}$

RANGE PARTITION (Google Kickstart 2022, Round C)

https://codingcompetitions.withgoogle.com/kickstart/round/00000000008cb4d1/0000000000b20deb

$1, 2, 3, \ldots\ldots, N$

N, X, Y

$$\gcd(X, Y) = 1$$

$\{1, 2, 5, 6\} : \{\_ \_ \_ \_\}$

$X : Y$

Can $\{1, 2, \ldots, N\}$ partitioned into 2 parts such that the ratio of sum of the parts are $X : Y$ for given N, X, Y. If YES, show a valid partition.

$N = 3$, $X = 1$, $Y = 2$

$\{1, 2, 3\} \longrightarrow \{2\}, \{1, 3\}$

$k = 2$

$$\frac{N(N+1)}{2} = (X+Y)\underline{k} \left\{ Xk \quad , \quad Yk \right\}$$

$$k = \frac{N(N+1)}{2(X+Y)}$$

$$(X+Y) \nmid \frac{N(N+1)}{2} \longrightarrow \text{NO.}$$

$$(X+Y) \mid \frac{N(N+1)}{2} \longrightarrow (\text{YES})$$

$$(Xk) = Z$$

$$1 \le Z < \frac{N(N+1)}{2}$$

- $1 \le Z \le N \longleftrightarrow \{Z\}, \ \{1, 2, \cdots N\} \setminus \{Z\}$
- $N < Z \le N + (N-1) \longleftrightarrow \{N, Z-N\}, \ \cdots$
- $N + (N-1) < Z \le N + (N-1) + (N-2)$

  $\{N, N-1, Z-N-(N-1)\}$

  $\vdots$

- $N + (N-1) + \cdots + (N-i) < Z \le N + (N-1) + \cdots + (N-i) + \{N-(i+1)\}$

Whenever Z is greater than the "last seen value", then take it.

for v from N to 1

if $Z \geq v$,

take v in your subset

$Z := Z - v$.

REARRANGEMENT INEQUALITY

$$\begin{cases} x_1 \leq x_2 \leq x_3 \leq \cdots \leq x_n \\ y_1 \leq y_2 \leq y_3 \leq \cdots \leq y_n \end{cases}$$

$$\underbrace{x_1 y_1 + x_2 y_2 + \cdots + x_n y_n \geq x_1 y_{\pi(1)} + x_2 y_{\pi(2)} + \cdots + x_n y_{\pi(n)}}$$

$$\geq x_1 y_n + x_2 y_{n-1} + \cdots + x_n y_1$$

π is a permutation of $\{1, \cdots, n\}$

$n = 4$, $\quad \{\underset{\pi(1)}{3}, \underset{\pi(2)}{2}, \underset{\pi(3)}{1}, \underset{\pi(4)}{4}\}$

$$x_1 y_1 + x_2 y_2 + x_3 y_3 + x_4 y_4 \geq x_1 y_3 + x_2 y_2 + x_3 y_1 + x_4 y_4$$

Proof :-

$$\sum_{i=1}^{n} x_i y_{\pi(i)} \text{ is maximum.}$$

$\{1, 2, 3, 6, 4, 5, 7\}$

i

$6 = j$

$4 = i$

$\exists i$

$\pi(i) \neq i$

$\forall i' < i, \quad \pi(i') = i'$

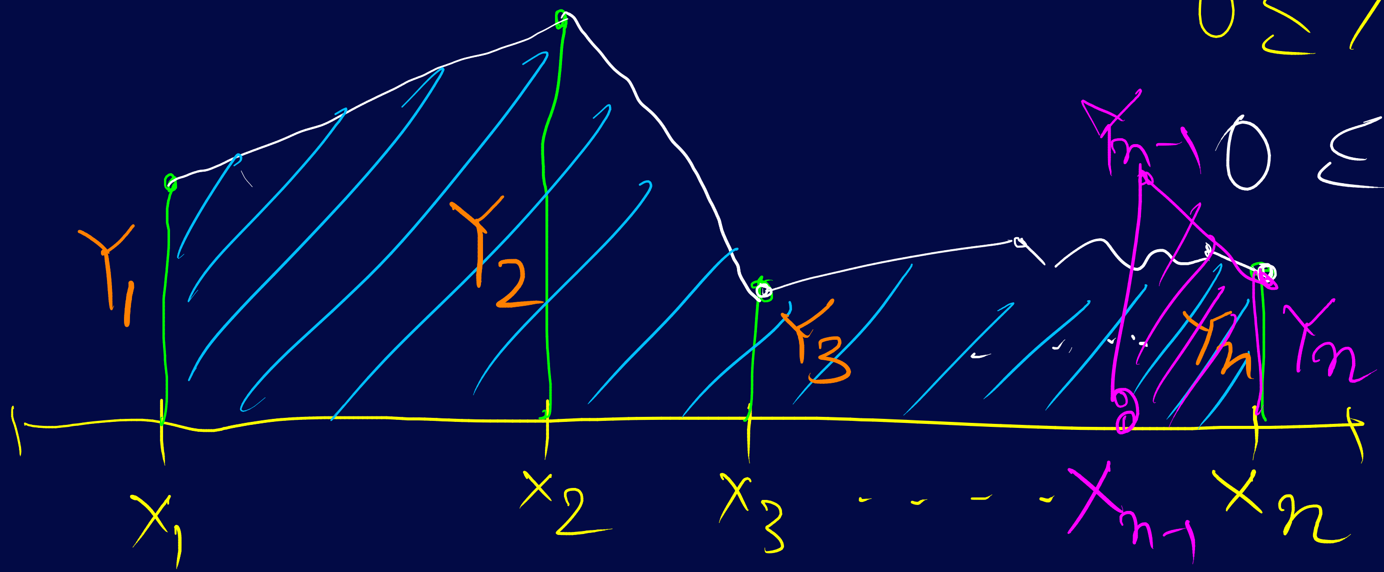
$\pi(i) = j > i$

$\pi(k) = i \qquad k > i$

$$\cdots + x_i y_{\pi(i)} + \cdots + x_k y_{\pi(k)} + \cdots$$

THE BIGGEST RESTAURANT (Codechef November Cook-Off 2019)

https://www.codechef.com/problems/BIGRES



$$0 \le X_1 < X_2 \cdots < X_n \le 2\times 10^9$$

$$0 \le H_1, H_2, \ldots, H_n \le 2\times 10^9$$

$$\underbrace{Y_1, Y_2, \ldots Y_n}_{H_1,\ H_2,\ \ldots\ H_n}$$

$$2\Delta = (X_2 - X_1)(Y_1 + Y_2) + (X_3 - X_2)(Y_2 + Y_3) + (X_4 - X_3)(Y_3 + Y_4) + \cdots + (X_n - X_{n-1})(Y_n + Y_{n-1})$$

$$= (X_2 - X_1)Y_1 + (X_3 - X_1)Y_2 + (X_4 - X_2)\cdot Y_3 + \cdots + (X_{n-2} - X_n)Y_{n-1} + (X_n - X_{n-1})Y_n$$

$$= \underbrace{(X_2 - X_1)}(Y_1) + \underbrace{(X_3 - X_1)}(Y_2) + \cdots + \underbrace{(X_n - X_{n-1})}(Y_n)$$

$$\left\{ \underline{X_2 - X_1},\ X_3 - X_1,\ \ldots,\ X_n - X_{n-2},\ \underline{X_n - X_{n-1}} \right\}$$

— Sort → this array → Take Dot Product.

Sort H

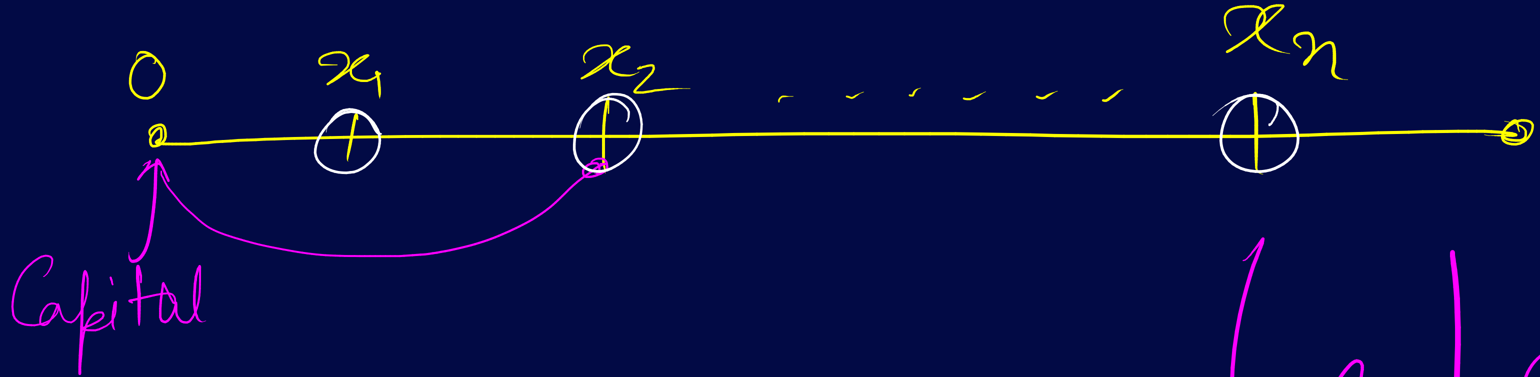
$$(X_3 - X_1) \le (X_4 - X_2) \le (X_2 - X_1) \le (X_n - X_{n-1}) \cdots$$

$$H'_1 \le H'_2 \le H'_3 \le H'_4$$

LINE EMPIRE (Codeforces Round #782, Division 2)

https://codeforces.com/problemset/problem/1659/C

$0 < x_1 < x_2 < \cdots < x_n$

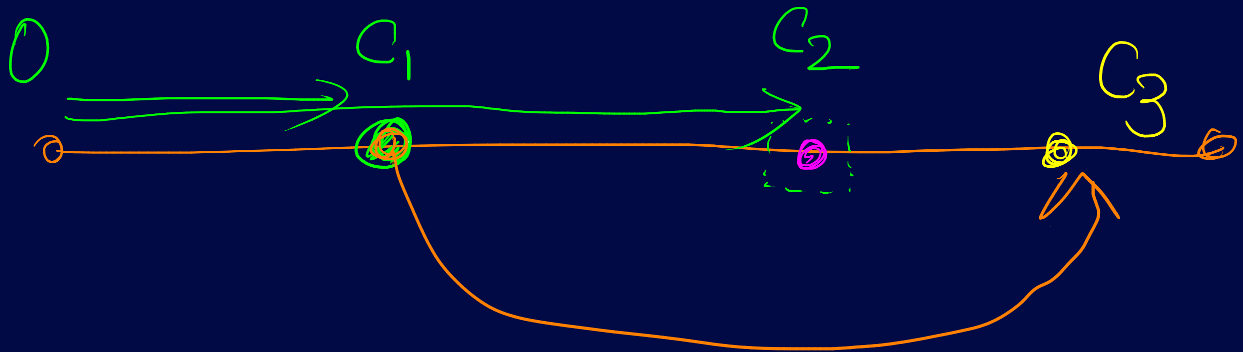


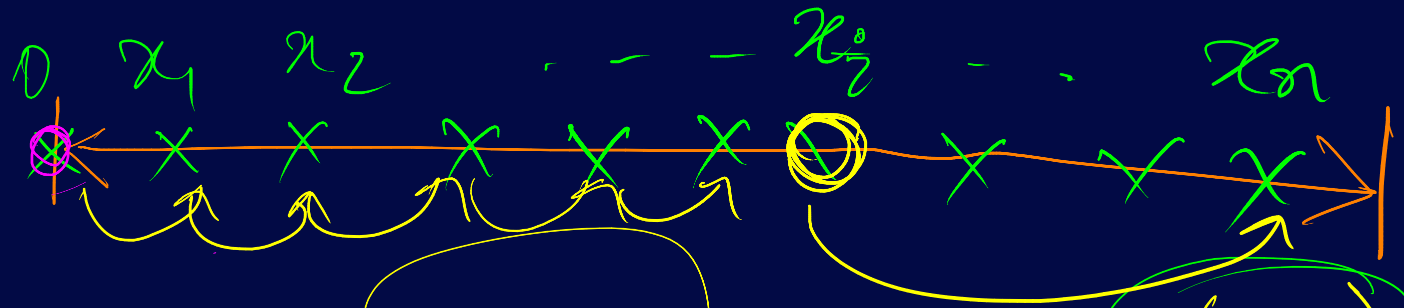
* $c_1 \longrightarrow c_2$

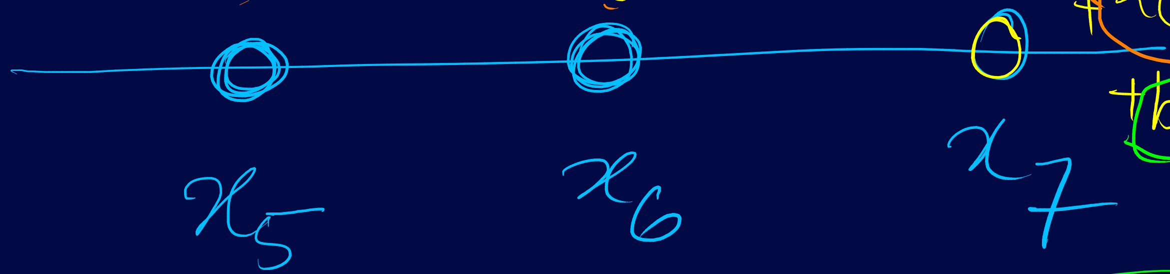
* $c_1 \longrightarrow c_3$

$$(a|c_2 - c_1|)$$

$$\{b|c_3 - c_1|\}$$







$x_5 < x_6 < x_7$

$a(x_6 - x_5)$ — Con. x_6

$+ a(x_7 - x_5)$ — Con. x_7

$+ b(x_6 - x_5)$ — more. x_6

$a(x_6 - x_5)$ — Con x_6

$b(x_6 - x_5)$ — more – x_6

$a(x_7 - x_6)$ — Con x_7

BINARY SEARCH REVISITED

AGRESSIVE COWS (USACO February 2005 Gold Division)

https://www.spoj.com/problems/AGGRCOW/

REFERENCES FOR EXPLORING FURTHER:

CLASSICAL PROBLEMS (May be Searched for in Google):

* Coin Change Problem (with coins 1, 2 and 5)
* Activity Scheduling Problem
* Fractional Knapsack Problem
* Egyptian Fractions
* Kadane's Algorithm

PROBLEMS FROM VARIOUS CODING PLATFORMS:

* https://www.codechef.com/problems/STONEARMY
* https://www.codechef.com/INOIPRAC/problems/INOI1201
* https://www.codechef.com/AM19MOS/problems/COLINT
* https://www.codechef.com/problems/ALIENIN
* https://codeforces.com/problemset/problem/1554/A
* https://codeforces.com/problemset/problem/1613/B
* https://codeforces.com/problemset/problem/1616/B
* https://codeforces.com/problemset/problem/1498/B
* https://codeforces.com/contest/1550/problem/C
* https://codeforces.com/problemset/problem/1554/D
* https://codeforces.com/contest/1684/problem/D
* https://www.hackerrank.com/contests/codenite-2021-round-1/challenges/open-iit-competitions/
* https://www.hackerrank.com/contests/goc-cdc-series-11/challenges/maximum-balance-sequence/
* https://cses.fi/problemset/task/1085
* https://oj.uz/problem/view/CEOI12_jobs

OTHER RESOURCES:

* https://codeforces.com/problemset?tags=greedy
* https://www.codechef.com/tags/problems/greedy
* https://www.geeksforgeeks.org/greedy-algorithms/
* https://usaco.guide/silver/binary-search?lang=cpp
* https://en.wikipedia.org/wiki/Zeckendorf%27s_theorem